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A Method for Calculating the Effect of Aircraft Maneuvers on Sonic Booms

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Sonic boom theories initiated by Friedman, Kane, and Sigalla have been extended to include the effects of diving and climbing aircraft on curved flight paths. It is shown that flight paths that are concave downward can cause focusing of the wave energy giving rise to high boom overpressures. The present theory indicates that undesirable focusing effects might be avoided by pitting aircraft acceleration against flight-path curvature.

Nomenclature

A	= ray-tube area
d	= distance between rays
D	= vertical distance below aircraft
g	= gravitational constant
k	= flight-path curvature
L	= lift force
l, m, n	= ray direction cosines relative to x, y, z
M	= Mach number
N	= unit vector in ray cone
R	= flight-path radius of curvature
S	= distance along ray
Δt	= time increment
V_a	= aircraft speed
$\dot{V}_a$	= aircraft acceleration
$\dot{V}_L$	= lifting acceleration
W	= aircraft weight
x, y, z	= ray coordinate system
X, Y, Z	= coordinates aligned with horizontal and vertical
x^*, y^*, z^*	= coordinates aligned with climbing or diving aircraft
μ	= Mach angle
ν	= ray inclination angle
$\Delta_1\nu$	= initial difference in slopes between two successive rays
$\Delta_2\nu$	= difference in ray slopes due to propagation effects
ϕ	= angle measured about aircraft axis
ψ	= angle between x^* and X axes
θ	= angle between x and X axes

Subscripts

h	= conditions at aircraft altitude

1. Introduction

THE present paper is a continuation of the work on sonic booms initiated in Ref. 1, where a technique was presented for describing atmospheric and also aircraft acceleration effects on sonic booms. However, the results were limited to the case of a straight horizontal flight path.

We will now extend these results to include aircraft maneuvers such as diving, climbing, and curved flight paths, with the restriction that they remain in a vertical plane. (It is clear that this restriction applies only to curved flight paths.) It will be shown that the maximum effects of flight-path curvature are in the plane of the curve. Therefore motions in a vertical plane most directly affect boom strength below the aircraft. If, however, one wanted to determine the effects of a horizontal curved flight path, the techniques developed in Secs. 2 and 3 can be applied.

Relative to the sonic boom problem, aircraft linear acceleration and curved flight paths (giving rise to centrifugal accelerations) lead to very similar results. That is, if the maneuver were such as to focus the wave energy at some point below the aircraft, a localized region of relatively high boom overpressure might occur at ground altitude. It will be shown, however, that for realistic flight patterns (both climbout acceleration and descent deceleration) it is possible to play acceleration against curvature to avoid or minimize these focusing effects.

In Sec. 2 the equations necessary for describing, in accordance with the theory of Ref. 1, climbing or diving flight paths will be presented. These will be extended in Sec. 3 to include curvature effects. It was found, subsequent to publication of Ref. 1, that the derivation given there (in Sec. 4) for the ray-tube area expression can be improved. A simpler and more correct expression will be derived in Sec. 4. The derivation will be sufficiently general so as to include both the aircraft acceleration and the flight-path curvature contributions. The results will be discussed in Sec. 5. Finally, in Sec. 6 a technique for obtaining the shock-ground intersection will be described.

Considerable use will be made of the results and equations in Ref. 1. Otherwise, rederivation of many previously obtained results would make this paper unnecessarily lengthy.

2. Ray Angle Geometry

In this section we will describe an extension to the general theory, which is given in Ref. 1. This extension will permit

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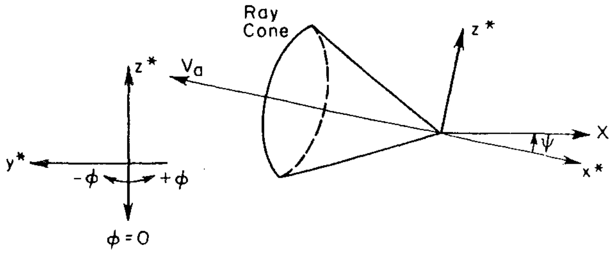

Fig. 1 Climbing aircraft coordinate system.

inclusion of flight paths that are curved, climbing, or diving. The results, however, are restricted to aircraft motions that are in a vertical plane. The contributions that arise from lateral motions can be determined by going through geometrical arguments very similar to those given below.

In order to include diving and climbing effects in the general theory, we first introduce a new coordinate system (x^*, y^*, z^*) (Fig. 1), in which x^* is tangent to the flight path, and the aircraft velocity is in the negative x^* direction; y^* is perpendicular to x^* and is horizontal; and z^* is perpendicular to y^* and x^* and points upward. The coordinates (x^*, y^*, z^*) are to form a right-handed system, and the aircraft is moving in the x^*, z^* plane. The angle ϕ will be used to identify any ray in the initial ray cone (Fig. 1). This angle has the same meaning as that defined in Fig. 1 of Ref. 1. Any unit vector in the initial ray cone (which is normal to the initial shock cone) has components relative to x^*, y^*, z^*, given in terms of Mach angle and rotation angle ϕ:

$$N_{x^*} = -\sin\mu \qquad N_{y^*} = -\cos\mu\sin\phi \qquad (1)$$
$$N_{z^*} = -\cos\mu\cos\phi$$

For climbing aircraft, the angle ψ between x^* and (horizontal) X is positive; for diving aircraft ψ is negative. That is, ψ is measured from the x^* to the X axes, positive in the counterclockwise direction. The components of unit vector N, given in Eq. (1), relative to X, Y, Z coordinates are

$$\left.\begin{aligned} N_X &= -\sin\mu\cos\psi - \cos\mu\cos\phi\sin\psi \\ N_Y &= -\cos\mu\sin\phi \\ N_Z &= \sin\mu\sin\psi - \cos\mu\cos\phi\cos\psi \end{aligned}\right\} \qquad (2)$$

We now want to construct the "ray coordinate system" x, y, z. Recalling [see discussion below Eq. (A11) of Ref. 1] that the angle θ is determined by requiring the ray to be initially in the x, z plane, we let l, m, and n be the x, y, and z direction cosines of the unit vector N. Rotating about the $Z = z$ axis (Fig. 2),

$$\left.\begin{aligned} l &= N_X\cos\theta + N_Y\sin\theta \\ m &= -N_X\sin\theta + N_Y\cos\theta \\ n &= N_Z \end{aligned}\right\} \qquad (3)$$

In order to have $m = 0$,

$$\left.\begin{aligned} \tan\theta &= \frac{N_Y}{N_X} = \frac{\cos\mu\sin\phi}{\sin\mu\cos\psi + \cos\mu\cos\phi\sin\psi} \\ \sin\theta &= \frac{-N_y}{(N_y^2 + N_x^2)^{1/2}} \qquad \cos\theta = \frac{-N_x}{(N_y^2 + N_x^2)^{1/2}} \end{aligned}\right\} \qquad (4)$$

Combining the results of Eqs. (3) and (4), the initial x-direction cosine of the ray is

$$l_h = -\cos\mu([\tan\mu\cos\psi + \cos\phi\sin\psi]^2 + \sin^2\phi)^{1/2} \qquad (5)$$

Equations (4) and (5) reduce to Eqs. (3.3) and (3.4) of Ref. 1 when the climb or dive angle ψ equals zero. Also, all the results of Ref. 1 can be applied using the more general definitions given in (4) and (5).

The results presented in this section make it possible to determine the ray locations and shock strengths for aircraft on straight climbing or diving flight paths. However, the technique used for determining the shock-ground intersection curve (Sec. 3.3 of Ref. 1) is inapplicable to the diving-climbing aircraft problem. This is because, for each instant along the flight path, the rays leaving the aircraft will have a different ground intersection curve. The main reason for this is that the aircraft altitude is continuously changing. The technique developed in Ref. 1 assumes that each set of ray ground intersections is the same and to know any one implies knowledge of all; therefore a shock-ground curve can be constructed although the rays that meet it have left the aircraft at different times. In Sec. 6 we will describe a method for determining an approximate shock-ground intersection.

It is not too difficult to include aircraft flight-path curvature in the analysis. This is of considerable importance for determining the ray-tube area used in shock strength computations. If the flight path is concave downward, two successive rays will be directed toward each other in a manner

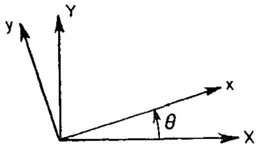

Fig. 2 Coordinate rotation.

very similar to that of an accelerating aircraft. At some point below the aircraft, the rays will converge leading, locally, to a high shock overpressure. The theory for this will be derived in the next section.

3. Flight-Path Curvature

It will be shown in Sec. 4 that perturbations to the ray inclination angle ν arise from two effects. The first, $\Delta_1\nu$, is the initial difference in the slopes of two successive rays due to aircraft motion. We will derive this contribution now (Fig. 3).

At the initial point A, the flight path is at an angle ψ with respect to the horizontal; at point B (assumed to be an infinitesimal distance from A), the angle has changed to $\psi + \Delta\psi$. Relative to a coordinate system at B, the components of a unit vector, identified by the angle $\bar{\phi}$, in the ray cone has components [Eq. (1)]

$$\bar{N}_{\bar{x}^*} = -\sin\mu \qquad \bar{N}_{\bar{y}^*} = -\cos\mu\sin\bar{\phi} \qquad (6)$$
$$\bar{N}_{\bar{z}^*} = -\cos\mu\cos\bar{\phi}$$

The bar notation is used to indicate variables at point B. In order to relate the components of $\bar{N}$ in (6) to the x^*, y^*, z^* system at A, we must rotate an amount $\Delta\psi$ about the y axis. Keeping terms of first order in $\Delta\psi$, i.e., $\cos\Delta\psi = 1$, $\sin\Delta\psi = \Delta\psi$,

$$\left.\begin{aligned} \bar{N}_{x^*} &= -\sin\mu - \cos\mu\cos\bar{\phi}\,\Delta\psi \\ \bar{N}_{y^*} &= -\cos\mu\sin\bar{\phi} \\ \bar{N}_{z^*} &= \sin\mu\Delta\psi - \cos\mu\cos\bar{\phi} \end{aligned}\right\} \qquad (7)$$

The two rays of interest are the intersections of a plane containing the flight path with the ray cones at A and B. This specification enables us to identify the angle $\bar{\phi}$ with the angle ϕ used in the previous section. At point A, we passed a plane through the x^* axis making an angle ϕ with the vertical (Fig. 1). At point B we passed a plane through the $\bar{x}^*$ axis making an angle $\bar{\phi}$ with the vertical. In order that the two rays, corresponding to ϕ at A and $\bar{\phi}$ at B, lie in the same plane, defined by ϕ, we must have

$$\frac{N_{y^*}}{N_{z^*}} = \tan\phi = \frac{\cos\mu\sin\bar{\phi}}{\cos\mu\cos\bar{\phi} - \sin\mu\Delta\psi} \qquad (8)$$

Letting $\bar{\phi} = \phi + \Delta\phi$ we obtain from (8)

$$\Delta\phi = -\sin\phi \tan\mu\Delta\psi \tag{9}$$

We can now determine the change in initial ray direction due to changes in both aircraft Mach number and flight-path slope. First we recall the identity [see Eq. (3.9) of Ref. 1] $\sin\nu = -l$, therefore $\Delta_1\nu$, the initial difference in slopes between two successive rays, is

$$\Delta_1\nu = -\sec\nu_h \Delta l_h \tag{10}$$

where l_h is the initial ray x- direction cosine. By using (5) we can determine

$$\Delta l_h = (\partial l_h/\partial\mu)(d\mu/dM)\Delta M + (\partial l_h/\partial\psi)\Delta\psi \tag{11}$$

$$\Delta M = \Delta V_a/a_h - V_a \Delta a/a_h^2 = \Delta W_a/a_h$$

where

$$\sin\mu = 1/M \qquad d\mu/dM = -[M(M^2 - 1)^{1/2}]^{-1}$$

and where

$$\begin{aligned} \Delta W_a &= \Delta V_a - M(da/dz)(dz/dt)\Delta t \\ &= \Delta V_a + M\beta\, V_a \sin\psi\ \Delta t \\ \beta &= \text{sound speed gradient, } -da/dz \\ &\cong 0.004 \text{ sec}^{-1} \text{ for altitudes 0–35,000 ft} \\ &\cong 0 \text{ for altitude 35,000–100,000 ft} \end{aligned}$$

After carrying out the differentiations in (11), using (9), and substituting the result in (10) we get

$$\Delta_1\nu = -\sec\nu_h \left[\frac{\Delta W_a \cos^2\nu_h}{M(M^2 - 1)a_h|l_h|} \left\{ 1 - \frac{\sin\psi}{\cos^2\nu_h} \times [\sin\psi - \cos\phi \cos\psi(M^2 - 1)^{1/2}] \right\} - \frac{\Delta\psi \cos\phi}{M} \times \{\cos\theta[\cos\psi(M^2 - 1)^{1/2} - \sin\psi\cos\phi] - \sin\theta\sin\phi\} \right] \tag{12}$$

Of the terms multiplying $\Delta\psi$, the one $\cos\theta \cos\psi\ (M^2 - 1)^{1/2}$ is largest. Therefore the coefficient of $\Delta\psi$ is positive, and a negative curvature $\Delta\psi < 0$ has the same effect as a positive acceleration. For an accelerating, climbing flight path the effects of a positive acceleration and curvature will offset each other. Similarly, for a decelerating, diving flight path the negative acceleration and curvature offset each other. This will be discussed further in Sec. 5.

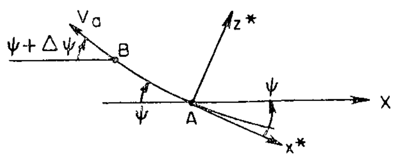

Fig. 3 Curved flight path.

4. Ray-Tube Area

The derivation of an expression for the ray-tube area, given in Ref. 1, has been improved. This derivation is as follows (Fig. 4): The change in distance between rays Δd due to a change in slope $\Delta\nu$ in a distance Δs along the ray is $\Delta d = \Delta\nu\ \Delta s$. Integrating this along the ray,

$$d = d_h + \int_0^s \Delta\nu ds \tag{13}$$

where

$$d_h = |V_a|\ \Delta t \cos\theta \cos\nu_h \tag{14}$$

The quantity $\Delta\nu$ will be considered to be made up of two parts

$$\Delta\nu = \Delta_1\nu + \Delta_2\nu \tag{15}$$

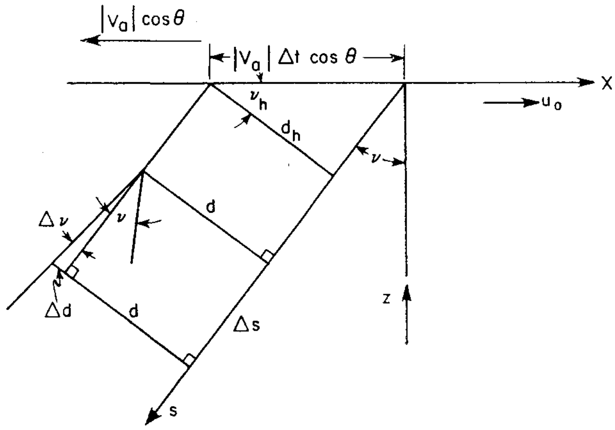

Fig. 4 Ray-tube area.

The first part $\Delta_1\nu$ is the initial difference between the slope of the two rays caused by aircraft motions. The second part $\Delta_2\nu$ is a change in slope due to the interrelation between the shock strength and ray-tube area.

In Eq. (12) $\Delta_1\nu$ is given, and variations along the ray, given by $\Delta_2\nu$, are determined from Eq. (3.9) of Ref. 1:

$$\sin\nu = V_s/(V_a \cos\theta + u_0)$$

This leads to

$$\Delta_2\nu = \tan\nu[(\Delta V_s/V_s) - \sin\nu(\Delta u_0/V_s)] \tag{16}$$

As in Ref. 1, Sec. 4, we let

$$A \sim zd \tag{17}$$

Combining Eqs. (12–17), we obtain, using $\dot{W}_a = \dot{V}_a + M\beta V_a \sin\psi$

$$A = z \sec\nu_h \left[1 - s \left\{ \frac{\dot{W}_a[M^2(M^2 - 1)]^{-1}}{a_h^2|l_h| \cos\theta} \times \left(1 - \frac{\sin\psi}{\cos^2\nu_h} [\sin\psi - \cos\psi \cos\phi(M^2 - 1)^{1/2}] \right) - k \frac{\cos\phi}{M \cos^2\nu_h} [\cos\psi(M^2 - 1)^{1/2} - \sin\psi \cos\phi - \tan\theta \sin\phi] \right\} + \frac{\sec\nu_h}{V_a \cos\theta} \int_h^z \tan\nu \left(\frac{dV_z}{dz} - \sin\nu \frac{du_0}{dz} \right) dz \right] \tag{18}$$

The term k is the rate of change of flight-path angle with respect to distance along the flight path, i.e., $k = d\psi/dtV_a$. This is, by definition, the flight-path curvature. The term $\cos\phi$ multiplying k indicates that the maximum effect of curvature is in the plane of the curve $\phi = 0$, below the aircraft. This is true, in general, for curvature in any plane containing the flight path.

5. Discussion of Results

The curvature can be related to aircraft dynamics as follows: Consider a curved flight path, with the (simplified) force diagram as shown in Fig. 5. Balancing the radial forces

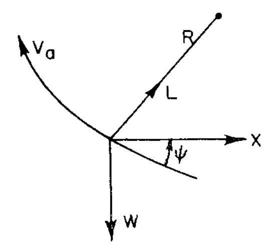

Fig. 5 Simplified force diagram: L = lift, W = weight, R = radius of curvature.

at the instantaneous center of curvature,

$$(W/g)(V_a^2/R) = L - W\cos\psi$$

Therefore,

$$k = 1/R = [(L - W\cos\psi)g]/WV_a^2 \qquad (19)$$

When lift L is greater than $W\cos\psi$, the aircraft is increasing its flight-path angle, and the curvature is positive; when L is smaller than $W\cos\psi$, the curvature is negative.

In order to get a better understanding of Eq. (18), we will consider an accelerating aircraft on a curved flight path. At the instant under consideration, the aircraft is moving horizontally ($\psi = 0$); also, for simplicity we consider the problem directly below the aircraft ($\phi = \theta = 0$). For this case, the first terms within the bracket on the right side of Eq. (18) can be written

$$1 - S\left\{\frac{\dot{V}_a}{a_h^2 M(M^2-1)} - \frac{kM}{(M^2-1)^{1/2}}\right\} \qquad (20)$$

Using (19) to determine k and setting (20) equal to zero, we solve for $D = S\cos\mu = S(M^2-1)^{1/2}/M$. Where D is vertical distance from the aircraft,

$$D = \frac{a_h^2(M^2-1)^{1.5}M^{-1}}{\dot{V}_a\sin\mu - \dot{V}_L\cos\mu} \qquad (21)$$

where $\dot{V}_L$ = lifting acceleration $= [(L - W)/W]\cdot g$.

Equation (21) (which is in agreement with Eq. 8 of Ref. 2) gives the distance from the aircraft where the ray-tube area will vanish (approximately) because of aircraft acceleration and/or flight-path curvature. It is easily seen that the denominator in (21) is just the projection along the ray, below the aircraft, of the two accelerations. It should be possible to adjust the Mach number, linear acceleration, and flight-path curvature in such a way as to keep D considerably larger than the aircraft altitude. When the aircraft is in a climbout-acceleration (descent-deceleration) mode, both $\dot{V}_a$ and $\dot{V}_L$ are positive (negative); it may therefore be possible to determine a flight plan that will cause D to be quite large. When D is greater than the aircraft altitude, the rays, or shock, will reach the ground before focusing, and hence the wave energy will be spread out over a large region.

6. Shock-Ground Intersection

When an aircraft is either climbing or diving, the shock-ground intersection curve varies with time. The problem is, therefore, basically different from the one in which the aircraft is flying horizontally. This latter problem is truly a steady-state situation, and the shock-ground intersection curve is invariant.

The shock-ground curve is the locus of disturbances that reach the ground simultaneously. By integrating the ray equations, we obtain the locus of disturbances that leave the aircraft at the same time. There are many ways to determine a shock curve when ray-curve data are known; however, the one described below seems to be comparatively simple and uses a minimum of effort.

Two points on the flight path are determined which are separated, in time, by an increment τ. Then, the ray-ground intersections are computed for each of these points. To be specific, assume data are determined for seven angles φ about the flight path. The following are then computed: seven ray-ground X, Y coordinates, seven ray travel times, and seven pressure jumps for each of the two points on the flight path. A mean ray travel time is then found simply by averaging the fourteen computed travel times, i.e.,

$$t_{\text{mean}} = \frac{1}{14}\sum_{i=1}^{7}(t_{Ai} + t_{Bi}) \qquad (22)$$

where t_{Ai} or t_{Bi} is the ray travel time from point A or B on the flight path corresponding to angle φ_i. Then, using this mean time, we determine by linear interpolation (or extrapolation) the corresponding X, Y coordinates and the pressure jump Δp as follows:

$$X_{i\,\text{mean}} = X_{Ai} + \frac{t_{Ai} - t_{\text{mean}}}{t_{Ai} - t_{Bi}}(X_{Bi} - X_{Ai}) \qquad (23)$$

with the identical formula being used with Y or Δp substituted for X.

The resulting coordinates are, approximately, the ground intersection points of disturbances (shock) arriving at the ground simultaneously. The pressure jumps are the pressure jumps across this shock.

It is recognized that the foregoing computation gives some hypothetical "mean" shock and its strength. This is simply intended as an aid in visualizing the ground-shock pattern. A computer program has been written which carries out, for the theory given here and in Ref. 1, the complete sonic boom computation. This program is described in Ref. 3.

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